Let $a, b \in \bar{\mathbb{Q}}$ be such that exactly one of $a$ and $b$ is an algebraic integer, and let $f_t(z) = z^2 + t$ be a family of quadratic polynomials parametrized by $t \in \bar{\mathbb{Q}}$. We prove that the set of all $t \in \bar{\mathbb{Q}}$ for which there exist $m, n \geq 0$ such that $f_t^m(a) = f_t^n(b)$ has bounded height. This is a special case of a more general result supporting a new bounded height conjecture in arithmetic dynamics. This is joint work with DeMarco, Ghioca, Krieger, Tucker, and Ye. (Received September 20, 2017)