

1135-11-1320      **Byungchul Cha\*** (cha@muhlenberg.edu), Muhlenberg College, 2400 W. Chew st, Allentown, PA 18104, and **Dong Han Kim**. *Berggren trees, Romik system and Lagrange Theorem*.

An old result of Berggren says that there exist three  $3 \times 3$  matrices  $M_1, M_2, M_3$  such that, for every triple  $(x, y, z)$  of positive coprime integers (with  $y$  even) satisfying  $Q(x, y, z) := x^2 + y^2 - z^2 = 0$ , there exists a unique sequence  $d_1, d_2, \dots, d_k$  with  $d_j \in \{1, 2, 3\}$  such that

$$(x, y, z)^\top = M_{d_1} \cdots M_{d_k} (3, 4, 5)^\top.$$

If  $y$  is odd, the same is true with  $(4, 3, 5)$  replacing  $(3, 4, 5)$ . That is, the set of all such triples  $(x, y, z)$  forms infinite ternary trees rooted at  $(3, 4, 5)$  and  $(4, 3, 5)$ . We study the generalizations of these trees arising from other quadratic forms  $Q(x, y, z)$ . Also, we investigate the properties of Romik's dynamical system, which closely resembles the classical dynamical system of continued fractions on the unit interval defined by Gauss map. In particular, we present an analogue of Lagrange Theorem for Romik system which concerns eventually periodic digit expansions for points over a real quadratic field. This work originated from the collaboration with Emily Nguyen ('16) and Brandon Tauber ('16), funded by Center for Undergraduate Research in Mathematics. (Received September 21, 2017)