Finding the four squares in Lagrange’s Theorem.

In 1986, Rabin and Shallit presented three random algorithms to compute, given a positive integer \( n \), integers \( X, Y, Z, W \) with \( X^2 + Y^2 + Z^2 + W^2 = n \). The fastest of the three has expected runtime \( \mathcal{O}(\log n)^2 \), but this runtime analysis assumes the truth of the Extended Riemann Hypothesis. The other two algorithms admit slightly worse runtime estimates but are unconditional, in the sense that no unproved hypotheses are used in the proof of correctness or the running-time analysis. In this talk we explain how to modify their algorithms to do better unconditionally. (Received September 24, 2017)