I refer to the function of Pythagoras that sends $n$ to $s(n)$, the sum of the divisors of $n$ that are less than $n$. A still-open conjecture of Catalan & Dickson: each orbit in the $s$-dynamical system (i.e., $n, s(n), s(s(n)), \ldots$) is bounded. Modeling such a sequence as a random geometric progression, Bosma & Kane showed that the average of $\log(s(n)/n)$ for $n$ even is a constant $\lambda < 0$ (and for $n$ odd, it’s $-\infty$). A new result: the average of $\log(s(s(n))/s(n))$ for $n$ even is also $\lambda$.

Pythagoras noted 2-cycles in the $s$-dynamical system, the so-called amicable numbers. It’s been known since 1981 that the reciprocal sum of the amicable numbers is finite, and in 2011 Bayless & Klyve showed this sum is $< 656,000,000$. Recently with Nguyen we improved the bound to 222.

I also report on some new results with Pollack & Thompson on fibers of $s$. (Received September 09, 2017)