If $M$ is a maximal ideal of height at least two in a Noetherian ring $R$, a well-known inequality asserts that $|R/M| + \aleph_0 \leq |\text{Spec } R| \leq |R| \leq |R/M|^{\aleph_0}$. Given that many naturally encountered local rings $R$ of dimension at least two satisfy $|\text{Spec } R| = |R|$, it is natural to wonder if equality can ever fail. Using a technique inspired by the works of Nagata and Heitmann, we prove that there are examples of local rings $R$ such that $|\text{Spec } R| < |R|$ in every Krull dimension. The technique, which may be of independent interest, essentially allows one to greatly enlarge a Noetherian ring while completely preserving its prime ideal structure. We conclude by showing that such curious examples of Noetherian rings with $|\text{Spec } R| < |R|$ contain Noetherian subrings $S$ such that $|S| = |\text{Spec } R|$ and $\text{Spec } S \cong \text{Spec } R$. (Received September 25, 2017)