Let $k$ be an algebraically closed field of characteristic 0, and let $f \in k[[x, y]]$ be the germ of an isolated plane curve singularity. We study the role of the singularity germ $f$ in the analysis of inflectionary behavior of curves specializing to a curve with a singularity cut out by $f$. We introduce a numerical function $m \mapsto AD_m(f)$, an invariant canonically associated to the isomorphism class of the singularity germ $f$, which arises as an error term in the problem of enumerating $m^{th}$-order inflection points in a 1-parameter family of curves acquiring a singular member with singularity given by $f = 0$. For an ordinary nodal singularity $f = xy$, we explicitly compute $AD_m(f) = \binom{m+1}{4}$, and we deduce as a corollary that $AD_m(f) \geq \mu_f \cdot \left(\begin{array}{c} m+1 \\ 4 \end{array}\right)$ for an arbitrary $f$, where $\mu_f$ is the Milnor number of $f$. The numerical function $m \mapsto AD_m(f) - \mu_f \cdot \left(\begin{array}{c} m+1 \\ 4 \end{array}\right)$ is thus also an invariant of the singularity type, and it measures the extent to which the singularity counts as an $m^{th}$-order inflection point. Our main results can be applied to address a broad range of enumerative questions concerning inflection points in families of curves. (Received September 16, 2017)