Let $\mathbb{H}^{m \times n}$ be the space of $m \times n$ matrices over $\mathbb{H}$, where $\mathbb{H}$ is the real quaternion algebra. Let $A_\phi$ be the $n \times m$ matrix obtained by applying $\phi$ entrywise to the transposed matrix $A^T$, where $A \in \mathbb{H}^{m \times n}$ and $\phi$ is a nonstandard involution of $\mathbb{H}$. We first give some properties of the Moore-Penrose inverse of the quaternion matrix $A_\phi$. Then we consider two systems of mixed pairs of quaternion matrix Sylvester equations $A_1X - YB_1 = C_1$, $A_2Z - YB_2 = C_2$ and $A_1X - YB_1 = C_1$, $A_2Y - ZB_2 = C_2$, where $Z$ is conditions for the existence of a solution $(X,Y,Z)$ to those systems in terms of the ranks and Moore-Penrose inverses of the given coefficient matrices will be presented. Moreover, the general solutions to these systems are explicitly given when they are solvable. (Received September 26, 2017)