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Kristin A. Camenga and **Patrick X Rault*** (rault@email.arizona.edu), 9040 S Rita Rd, Ste 2260, Tucson, AZ 85747. *A Proof of a Generalized Lax Conjecture for Numerical Ranges*. Preliminary report.

Let A denote an $n \times n$ unitarily irreducible complex matrix. Let H_1 and iH_2 be the hermitian and skew-hermitian parts of A , and define the polynomial $F_A(x, y, t) := \det(xH_1 + yH_2 + tI)$. We define $\Gamma_F : F(x, y, t) = 0$ in $\mathbb{P}^2(\mathbb{R})$, whose dual $\Gamma_{\hat{F}}$ is called the boundary generating curve. The convex hull of the latter is the numerical range of A , denoted $W(A)$. This process thus gives a method for going from A to $W(A)$. Lax conjectured in 1958 that that we can also do the reverse: for a certain type of curve \mathcal{C} , there exist symmetric matrices B and C for which $G(x, y, t) := \det(xB + yC + tI)$ satisfies $\mathcal{C} = \Gamma_{\hat{G}}$. This conjecture was proved in 2005 by Lewis, Parrilo, and Ramana. In 2012, Helton and Spitkovsky completed the connection by proving that there exists a symmetric matrix S for which $F_A = F_S$; in particular $W(A) = W(S)$. In this talk we give a concise proof of the following generalization: For any factor g of F_A , there exists a symmetric matrix T for which $F_T = g$. We will analyze examples of these factors and how the curves they define can intersect.

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