The degenerate two boundary Hecke algebra $H_d$ is generated by the symmetric group on $d$ letters and polynomial rings subject to further relations. It acts on the tensor space $M \otimes N \otimes V^\otimes d$, where $M$ and $N$ are irreducible polynomial representations of the Lie superalgebra $\mathfrak{gl}(n|m)$ whose highest weights are represented by rectangular Young diagrams, and this action commutes with that of $\mathfrak{gl}(n|m)$. As a module for the centralizer of $\mathfrak{gl}(n|m)$, $M \otimes N \otimes V^\otimes d$ decomposes into irreducible modules labeled by hook Young diagrams, and a basis is given via Young tableaux where the polynomial generators act by explicit combinatorial eigenvalues. These modules remain irreducible when restricted to the action of $H_d$, and provide a class of irreducible representations for $H_d$. This construction generalizes results in the $\mathfrak{gl}(n)$ case by Zajj Daugherty (2010.) (Received September 07, 2017)