Networks can be combined in many ways including by overlaying one on top of another or sitting one next to another. We encode these two ways of combining networks as a specific kind of functor and prove that the application of a novel general construction to these functors results in typed operads. The class of operads we construct—which we call network operads—contains a wealth of examples whose many to one operations serve as a syntax for designing complex networks by composing simpler networks. We give examples of concrete ways to compose networks with our setup by matching specific kinds of networks to actions of network operads. Remarkably network operads can provide a unified treatment of the structural design and behavioral tasking of dynamic networks. (Received September 26, 2017)