Quasi-Möbius maps on the Morse boundary.

The Morse boundary, $\partial_*X$ is a quasi-isometry invariant for proper metric spaces, and thus can be used to distinguish non quasi-isometric groups. An interesting question to explore is, when can the Morse boundary serve as a tool to show two groups are quasi-isometric? In analogy with the hyperbolic manifold case, it is possible to define quasi-Möbius structures on the Morse boundary. Quasi-Möbius maps give an answer to this question. Given two proper metric spaces with non-empty Morse boundary, say $X$ and $Y$, there is a quasi-Möbius map $h : \partial_*X \to \partial_*Y$ if and only if $f$ extends to a quasi-isometry $f : X \to Y$. (Received September 24, 2017)