Moving from Simple Analysis to Research through Exploration.

The focus of this talk is to start with a rather simple analysis problem and explore the different directions that it can take us, in the process demonstrating how these kinds of explorations can bring us to the edge of research-level problems. Suppose that for $f : \mathbb{R} \to \mathbb{R}$, we have $|f'(x)| \leq \lambda |f(x)|$ for some $\lambda > 0$ and for all $x$; $f$ is continuous and differentiable; and $f(0) = 0$. We will show that $f(x) = 0$ everywhere, and after exploring the one-dimensional version, we will generalize the problem to $f : \mathbb{R}^n \to \mathbb{R}^m$ and to the case in which $f$ is merely Lipschitz continuous (which implies that $f$ is only differentiable almost everywhere). Lastly, we will examine this problem from a more intuitive geometric perspective and briefly discuss our current research direction stemming from this simple problem. (Received September 22, 2017)