A new method of construction of algebro-geometric solutions of rank two Schlesinger systems is presented. For an elliptic curve represented as a ramified double covering of $\mathbb{CP}^1$, a meromorphic differential is constructed with the following property: the common projection of its two zeros on the base of the covering, regarded as a function of the only moving branch point of the covering, is a solution of a Painlevé VI equation. This differential provides an invariant formulation of a classical Okamoto transformation for the Painlevé VI equations. A generalization of this differential to hyperelliptic curves is also constructed. The corresponding solutions of the rank two Schlesinger systems associated with elliptic and hyperelliptic curves are constructed in terms of these differentials. The initial data for the construction of the meromorphic differentials include a point in the Jacobian of the curve, under the assumption that this point has non-variable coordinates with respect to the lattice of the Jacobian while the branch points vary. This method is motivated by an observation of Hitchin, who related the Poncelet polygons to algebraic solutions of a Painlevé VI equation. The research has been partially supported by the NSF grant 1444147. (Received September 25, 2017)