We consider positive solutions to equations of the form
\[
\begin{cases}
-\Delta u = \lambda u(1-u), & x \in \Omega, \\
\frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda} g(u) u = 0, & x \in \partial \Omega,
\end{cases}
\]
where \(\lambda, \gamma > 0\) are parameters, \(g : [0, 1] \to [0, \infty)\) is a nonlinear function, and \(\Omega\) is a bounded domain in \(\mathbb{R}^N, N \geq 1\). Such problems arise in the study of population dynamics in a habitat \(\Omega\) when the population exhibits nonlinear density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness, and multiplicity of positive steady states) as the patch size (\(\lambda\)) and hostility of the outside matrix (\(\gamma\)) vary. We obtain our results via the method of sub-super solutions. (Received August 25, 2017)