We propose a modification of the standard inverse scattering transform for the focusing nonlinear Schrödinger equation (also other equations by natural generalization). The purpose is to deal with arbitrary-order poles and potentially severe spectral singularities in a simple and unified way. As an application, we use the modified transform to place the Peregrine solution and related “rogue wave” solutions in an inverse-scattering context for the first time. This allows one to directly study the stability of such solutions. The modified transform method also allows rogue waves to be generated on top of other structures by elementary Darboux transformations, rather than the generalized Darboux transformations in the literature. (Received September 26, 2017)