We show that the total area of two distinct Gaussian curvature 1 surfaces with the same conformal factor on the boundary, which are also conformal to the Euclidean unit disk, must be at least $4\pi$. In other words, the areas of these surfaces must cover the whole unit sphere after a proper rearrangement. We refer to this lower bound of total areas as the Sphere Covering Inequality. This inequality and its generalizations are applied to a number of open problems related to Moser-Trudinger type inequalities, mean field equations and Onsager vortices, etc, and yield optimal results. In particular we confirm the best constant of a Moser-Trudinger type inequality conjectured by A. Chang and P. Yang in 1987. This is a joint work Changfeng Gui. (Received September 26, 2017)