A new hierarchy of Banach spaces $T_k(d, \theta)$, $k$ any positive integer, is constructed using barriers in high dimensional Ellentuck spaces (introduced by Dobrinen) following the classical framework under which a Tsirelson type norm is defined from a barrier in the Ellentuck space.

It is shown that these spaces contain arbitrarily large copies of $\ell_\infty^n$, with the bound constant for all $n$. These spaces are $\ell_p$-saturated, in fact forming natural extensions of the $\ell_p$ spaces. They are not isomorphic to each other, but form a strict hierarchy: For fixed $d$ and $\theta$, for any $j < k$, the space $T_j(d, \theta)$ embeds isometrically into $T_k(d, \theta)$. (Received September 25, 2017)