Mathematical billiards is the study of the possible trajectories of a particle that bounces around some domain (e.g. a polygon or an ellipse) with elastic reflections. Many recent discoveries about the dynamics of billiards in polygons have utilized the connection between these billiard tables and geometric structures called translation surfaces, which can be thought of as collections of polygons with sides glued in parallel pairs or as Riemann surfaces with some extra structure. If we relax the structure on these translation surfaces just a bit, we get dilation surfaces. In this talk, we will survey the trajectory that we take to get from mathematical billiards to dilation surfaces, and say a bit about what is known and still unknown about the geometry and dynamics of dilation surfaces. (Received September 25, 2017)