While the moduli space $\mathcal{M}_G(\Sigma)$ of flat connections on a $G$-principal bundle over a surface $\Sigma$ has been extensively studied, the case of a higher dimensional base $M$ remains largely unexplored. Given a Lefschetz symplectic form on $M$, there is an induced symplectic structure on the moduli space $\mathcal{M}_G(M)$. We will show that, under the action of the gauge group, $\mathcal{M}_G(M)$ is a generalized symplectic quotient of the space of all $G$-connections over $M$, endowed with a natural vector-valued symplectic form. For special cases of $M$ and $G$, we also obtain a description of the topology of $\mathcal{M}_G(M)$, as well as an analytic expression for the symplectic volume. (Received September 26, 2017)