In the many attempts to prove Euclid’s Fifth Postulate, several equivalently self-evident assumptions were stated. Perhaps the most famous is Playfair’s Axiom: Given a straight line and a point not on it there exists a unique parallel line through the point.

In this report we will analyze the necessity of Euclidean space—even without the isotropy assumption— with the classical Fifth postulate and with Playfair’s Axiom. For the latter, we will discuss the seemingly weaker assumption of total curvature.

In particular, we will study the implications at infinity both of the parallel postulate, of volume growth, and of the integrability of the Gaussian Curvature for complete metrics on $\mathbb{R}^2$. (Received September 26, 2017)