Greg Yang* (gregyang@microsoft.com). A Homological Theory of Functions.

Classical learning theory studies whether we can learn an unknown function within a known class of functions, given access to example input/output pairs. I will show that, surprisingly, the learnability of a class is controlled by its “homology” — specifically, I will associate to each class a simplicial complex, and I will prove that the VC dimension of the class is bounded above by, roughly speaking, the highest dimension of any nonvanishing homology group of this complex, with equality occurring for many common classes occurring in computer science or algebra. If time permits, I will also mention applications of this homological construction in computational complexity lower bounds and a “homological Farkas lemma” that characterizes, via homology, when a linear subspace intersects the positive cone. (Received September 23, 2017)