The group $\text{PSL}_2(\mathcal{O})$, where $\mathcal{O}$ is an order in Hamilton’s quaternions, acts discretely by Möbius transformations on hyperbolic 4- or 5-space (depending on the definition used for $\text{PSL}_2$), giving rise to hyperbolic 4- or 5-manifolds. We introduce a generalization of this where $\mathcal{O}$ is instead an order in a definite quaternion algebra over a real quadratic number field, and the action now occurs on a product of two copies of hyperbolic 4- or 5-space via a Galois twist (analogous to the classical Hilbert-Blumenthal surfaces), giving rise to 8- or 10-dimensional manifolds.

We present a fundamental domain for the cusp of such a manifold, which facilitates the study of its topology and dynamics. We also discuss analogous new fundamental domains for classical Hilbert-Blumenthal surfaces that were developed concurrently by the authors with the intention of generalizing to the quaternionic version, and which serve nicely to build intuition for the higher-dimensional setting. (Received August 03, 2017)