Consider a birth-death chain on state space \( S = \{0, 1, 2, \ldots, H\} \) where \( H \in \mathbb{N} \) with alternating birth probabilities \( c_1, c_2, c_1, c_2, \ldots \) and alternating death probabilities \( a_1, a_2, a_1, a_2, \ldots \) where \( 0 < c_i < 1, 0 < a_i < 1 \) and \( a_i + c_i \leq 1 \) for \( i = 1, 2 \). Assume \( a_1c_2 = a_2c_1 \) or \( a_1c_1 = a_2c_2 \). Suppose \( P \) is the one-step transition probability matrix associated with this birth-death chain and let \( P^* \) be the one-step transition probability matrix of the corresponding dual birth-death chain.

Conclusions:
1) The set of eigenvalues of \( P \) and \( P^* \) are the same and are determined as a function of \( H \).
2) \( P^n \) and \( (P^*)^n \) can be determined exactly for \( n \in \mathbb{N} \).
3) Birth-death chains that have the same set of eigenvalues are identified.
4) Related results for birth-death processes and non-birth-death chains are discussed. (Received September 23, 2017)