Markov Chain Monte Carlo (MCMC) is the prevalent approach for sampling posterior distributions in Bayesian inverse problems. For inverse problems with many measurements, MCMC methods are prohibitively expensive since overall $10^6 - 10^8$ large-scale linear systems need to be solved to evaluate the likelihood and explore the parameter space. We propose several different methods of reducing the cost of the likelihood evaluations using techniques from randomized matrix methods. Tradeoffs between computational costs and statistical efficiency will be discussed. The performance of the proposed samplers will be demonstrated on 2D model problems from Diffuse Optical Tomography. (Received September 24, 2017)