The modeling challenges arising when the problem domain has small supported holes in it are considered through a representative membrane problem. Such problems are sometimes modeled intuitively in engineering practice by taking the limiting case of holes with zero radius. This intuitive model is incorrect, since it has no mathematical solution. It is demonstrated, however, that finite element approximations based on it can still satisfy verification tests and appear to converge, leading to erroneous recovery of quantities of interest. This points to the need for an alternate approach where the holes of finite radius are properly incorporated in the modeling, and robustness with respect to the radius is maintained. To this end, a computational method is presented which combines analytic knowledge of the solution singularities with finite element approximation of its smooth components. Theoretical and numerical results are provided, establishing the efficacy and robustness of the method in extracting quantities of interest. (Received September 26, 2017)