Bounds on polynomial eigenvalues are useful, e.g., for their computation by iterative methods, when computing pseudospectra, or, especially, in the analysis of engineering problems. We propose to refine and then generalize apparently little known - but useful - results on bounds for the zeros of scalar polynomials, i.e., polynomials with complex coefficients, to the matrix polynomials appearing in polynomial eigenvalue problems. These eigenvalues are much more difficult to compute than polynomial zeros, making bounds on such eigenvalues more valuable. Contrary to almost all existing bounds, our eigenvalue localization results can be further improved iteratively to achieve significant improvements over existing bounds. Furthermore, we obtain results for a large class of polynomial bases that includes - but is not limited to - all classical orthogonal bases: Hermite, Legendre, Chebyshev, etc. We present several applications from the engineering literature. (Received September 14, 2017)