Dynamics of Large Boson Systems with Attractive Interaction and a Derivation of the Cubic Focusing NLS in $\mathbb{R}^3$.

We consider a $N$-particle Boson system with two-body interaction $N^{3\beta-1}v(N^\beta x)$ where $v \in C_0^\infty$ for some range of $\beta$. We extend the results of Grillakis et al. in Comm. Math. Phys., (2013) and Kuz in arXiv:1511.00487 regarding second-order correction to mean-field evolution of systems with repulsive interaction to the case of attractive interaction for $0 < \beta < \frac{1}{2}$.

The two key ingredients used to extend to this case of attractive interaction are the proofs of the uniform global well-posedness of solutions to a family of Hartree-type equations and the corresponding $L^\infty$-decay estimates on the solutions. Inspired by the recent works Pickl J. Stat. Phys. (2010), Chen et al. in Theo. and Math. Phys., (2013) and Chen et al. in Arch. Ration. Mech. Anal. (2016), we also provide both a derivation of the focusing cubic nonlinear Schrödinger equation (NLS) in 3D from the many-body Boson system and its rate of convergence toward mean field. (Received September 26, 2017)