Mathematicians’ construction of meaning for derivatives and integrals of complex-valued functions.

We engaged five research mathematicians in describing their images of differentiation and integration for functions of complex variables. Our data collection and analysis focused on characterizing these experts’ movement between domains of thought ranging from real world experience to formal mathematical theory. The mathematicians relied heavily on direct application of concepts from differentiation of real-valued functions and employed rotation and stretching as a local linear description of the action of the function. They employed reasoning about real-valued line integrals to generalize to complex functions, but several struggled to interpret what was being accumulated in this case. In some cases, the experts’ fixation on particular features created barriers to progress developing their geometric reasoning. All of the participants began reasoning about the tasks by invoking symbolic or formal reasoning. Only later did some of the mathematicians invoke corresponding geometric interpretations, often carefully crafted to represent features of their formal reasoning. We use these examples to explore the implications for instruction of attending to the interplay of concrete and formal mathematical reasoning, rather than presuming meaning develops only from the former to the latter. (Received September 20, 2017)