Markov ordering conjectures. Preliminary report.

A Markov number is a number in the triple \((x, y, z)\) of positive integer solutions to the Diophantine equation \(x^2 + y^2 + z^2 = 3xyz\). Markov numbers are a classical topic in number theory related to many areas of mathematics such as combinatorics and cluster algebras. Markov numbers are related to cluster algebras by Markov snake graphs, where a Markov snake graph is the snake graph of a cluster variable of the once punctured torus. The number of perfect matchings of a Markov snake graph, given by the numerator of the associated continued fraction, is a Markov number. In this talk, we discuss three conjectures given in Martin Aigner’s book [A] that provide an ordering on the Markov numbers \(m_{p/q}\) for a fixed numerator \(p\), fixed denominator \(q\) and a fixed sum \(p + q\).

[A] M. Aigner, Markov’s theorem and 100 years of the uniqueness conjecture, Springer 2010 (Received September 19, 2017)