Mazur and Tate introduced a $p$-adic $\sigma$ function defined on the kernel of reduction of an ordinary elliptic curve defined over a complete discrete valuation domain of residual characteristic $p > 2$, which they used to compute $p$-adic local heights. The logarithmic derivative of this function is a variant of a Weierstrass $\zeta$ function. From the perspective of $p$-adic integrality, the $\zeta$ function is the more natural object. For $p > 3$ we produce a $\zeta$ function as a Laurent series from a limit of mod $p^n$ objects on a universal ordinary Weierstrass model, deducing the interality of the $\sigma$ function via an explicit comparison between the universal curve and its quotient by the canonical $p$-torsion subgroup. (Received September 26, 2017)