In 1939, Frucht proved that for any finite group $G$, there exists a graph $\Gamma$ such that the automorphism group of $\Gamma$ is isomorphic to $G$. Naturally, this result gave rise to numerous extremal problems in graph theory. For instance, vertex-minimal graphs with a prescribed automorphism group are the subject of prior research by numerous authors. In this talk, we will discuss our proof of a conjecture made in 1980 by Marušič on the order of vertex-minimal planar graphs with cyclic symmetry of even order. Our proof completes a theorem giving the order of all vertex-minimal planar graphs with cyclic automorphism groups. We will also discuss further our proof regarding the order of vertex-minimal planar graphs with dihedral symmetry. (Received September 24, 2018)