Julia F. Knight* (knight.1@nd.edu), Karen Lange and Reed Solomon. Roots of polynomials in fields of generalized power series.

Let $K$ be an algebraically closed field of characteristic 0. Newton and Puiseux showed that the field $K\{\{t\}\}$ of Puiseux series is algebraically closed. Maclane showed that for a divisible Abelian group $G$, the field $K((G))$ of Hahn series is algebraically closed. Puiseux series have length at most $\omega$. For a given polynomial $p(x)$, Newton’s method for finding roots does not look computable. However, guessing at the non-computable bits, we get a uniform effective procedure that, when applied to any $K$ and a non-constant polynomial $p(x)$ over $K\{\{t\}\}$, yields a root. Hahn series have ordinal length. We can show that if $p(x)$ is a polynomial and $\gamma$ is a limit ordinal greater than the lengths of all coefficients in $p(x)$, then the roots all have length less than $\omega^\omega\gamma$. At least for countable ordinals $\gamma$, this is sharp. We would like to measure, in terms of the usual hierarchies from computability, the complexity of the process that, for a computable ordinal $\alpha$, given $K$, $G$, and a polynomial $p(x)$ over $K((G))$, either produces $r_\alpha$ of length $\alpha$ that is an initial segment of a root, or else determines a root $r$ of length less than $\alpha$. (Received September 18, 2018)