We first mention two algorithms for a certain sequence of nonnegative integers, one which calculates in \((\mathbb{Z}, +)\) in conjunction with the counting operator \# and the exponential substitution, and applies to any positive integer input. The other algorithm calculates in \((\mathbb{Z}, +, \cdot)\), and is more efficient when the input is a power of 2.

Next, let \(F\) be an ordered field, \(D\) a maximal discrete subring of \(F\), and \(G\) a maximal discrete additive subgroup of \(F\). We point out that although there are examples where \(F\) has elements of infinite distance to \(D\), it can never realize any gaps of \(G\). For countable \(F\), the subgroup \(G\) can be constructed \(\Delta^0_2\) relative to \(F\).

Finally we consider some nonstandard models \(M\) of weak arithmetic which have \(\mathbb{Z}\) as an additive direct summand. We present functions \(f, g : M \to M\) whose value at a sum minus sum of values is always 0 or 1 yet for some \(x, y, u, v \in M_{\geq 1}\), \(f(xy) < xf(y)\) and \(g(uv) > ug(v) + u - 1\). (Received September 21, 2018)