Many cliques with few edges. Preliminary report.

The problem of maximizing the number of cliques has been studied within several classes of graphs. For example, among graphs on $n$ vertices with clique number at most $r$, the Turán graph $T_r(n)$ maximizes the number of copies of $K_t$ for each size $t$. Among graphs on $m$ edges, the colex graph $C(m)$ maximizes the number of $K_t$'s for each size $t$.

In recent years, much progress has been made on the problem of maximizing the number of cliques among graphs with $n$ vertices and maximum degree at most $r$. The graph $aK_{r+1} \cup K_b$, where $n = a(r + 1) + b$ and $0 \leq b \leq r$, was shown to maximize the total number of cliques, and is conjectured to maximize the number of $K_t$'s for $t \geq 3$. This conjecture has been proven in significant cases.

In this talk, we discuss the edge analogue of this problem: which graphs with $m$ edges and maximum degree at most $r$ have the maximum number of cliques? We prove in some cases that the extremal graphs again contain as many disjoint copies of $K_{r+1}$ as can fit, with the leftovers in another component. In the edge analogue, these remaining edges form a colex graph. (Received September 24, 2018)