The $q,t$-Catalan numbers $C_n(q,t)$ are polynomials in $q$ and $t$ that reduce to the ordinary Catalan numbers when $q = t = 1$. These polynomials have important connections to representation theory, algebraic geometry, and symmetric functions. Work of Garsia, Haglund, and Haiman has given us combinatorial formulas for $C_n(q,t)$ as sums of Dyck lattice paths weighted by area and dinv. This talk continues an ongoing quest for a bijective proof of the symmetry property $C_n(q,t) = C_n(t,q)$.

We conjecture some structural decompositions of Dyck objects into infinite chains that can be paired up to prove the symmetry of some coefficients in $C_n(q,t)$. The chains are built from certain initial objects by applying an operator that increases dinv by 1 and reduces area by 1. A remarkable feature of these chains is that they do not depend on $n$ but explain the joint symmetry for all $n$ simultaneously. The chain construction leads to a combinatorial proof that for $0 \leq k \leq 9$ and all $n$, the terms in $C_n(q,t)$ of total degree \( \binom{n}{2} - k \) obey the required symmetry property. (Received September 11, 2018)