Melanie Ferreri* (fermj15@wfu.edu) and Jacob Liddy (liddyjacob@gmail.com). Ramsey Problems for Cycles versus \( K_5 \).

For graphs \( F, G, \) and \( H \), if all red-blue edge colorings of \( F \) contain either red \( G \) or blue \( H \) as a subgraph, then we write \( F \rightarrow (G, H) \). The Ramsey number for graphs \( G \) and \( H \), denoted \( R(G, H) \), is the smallest integer \( s \) such that \( K_s \rightarrow (G, H) \). It is known that \( R(C_n, K_5) = 4n - 3 \) for \( n \geq 5 \). We prove that for all \( n \geq 5 \), any graph on \( 4n - 4 \) vertices which does not contain \( C_n \) or an independent set of order 5 contains \( 4K_{n-1} \), and thus we characterize all Ramsey-critical graphs for \( C_n \) versus \( K_5 \). The graph \( K_{s-1} \uplus K_{1,t} \) is constructed by adding a vertex to \( K_{s-1} \) and joining it to \( t \) of its vertices. The star-critical Ramsey number \( r_*(G, H) \) is defined as the minimum \( t \) such that \( K_{s-1} \uplus K_{1,t} \rightarrow (G, H) \), where \( s = R(G, H) \). Values of \( r_*(C_n, K_m) \) are known for \( m \in \{3, 4\} \). In this work, we extend this to \( m = 5 \) and some cases for \( m = 6 \), and we present computational proofs of small cases and a computer-free proof of the general result for \( n \geq 8 \) and \( m = 5 \). We also compile a survey of known star-critical Ramsey numbers involving simple graphs such as cycles, paths, and fans. (Received July 28, 2018)