Formulas for Chebotarev densities of Galois extensions of number fields.

We generalize the Chebotarev density formulas of Dawsey (2017) and Alladi (1977) to the setting of arbitrary finite Galois extensions of number fields $L/K$. In particular, if $C \subset G = \text{Gal}(L/K)$ is a conjugacy class, then we establish that the Chebotarev density is the following limit of partial sums of ideals of $K$:

$$- \lim_{X \to \infty} \frac{1}{|G|} \sum_{\substack{2 \leq N(I) \leq X \\ I \in S(L/K; C)}} \frac{\mu(K)(I)}{N(I)} = \frac{|C|}{|G|},$$

where $\mu(K)(I)$ denotes the generalized Möbius function and $S(L/K; C)$ is the set of ideals $I \subset \mathcal{O}_K$ such that $I$ has a unique prime divisor $p$ of minimal norm and the Artin symbol $\left(\frac{L/K}{p}\right)$ is $C$. To obtain this formula, we generalize several results from classical analytic number theory, as well as Alladi’s concept of duality for minimal and maximal prime divisors, to the setting of ideals in number fields. (Received September 18, 2018)