Let $E$ be an elliptic curve defined over $\mathbb{Q}$ without complex multiplication. For each prime $\ell$, there is a representation $\rho_{E,\ell} : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$ that describes the Galois action on the $\ell$-torsion points of $E$. This representation is called the mod $\ell$ image of Galois.

In this talk, I will discuss what happens when one considers composite level images of Galois. In particular, I will introduce composite level modular curves whose rational points classify elliptic curves over $\mathbb{Q}$ with simultaneously non-surjective, composite image of Galois. I will also describe techniques used to provably find the rational points on these curves, which yield results concerning when composite level images of Galois occur.

Finally, I will give an application of our results to the study of entanglement fields and present non-CM elliptic curves with peculiar division fields. Some of the results I will talk about are joint work with Catalina Camacho, Wanlin Li, Jack Petok, and David Zureick-Brown. (Received September 25, 2018)