Generalizing the Three Gap Theorem.

Place the tip of a knife at the center of a circular cake and score the cake from center to edge. Rotate the cake under the knife and score it again after every rotation by angle $\theta$ until you have marks at $\theta, 2\theta, \ldots, N\theta$ for some fixed number $N$. After marking the cake this way, slice it up along those marks. No matter your $N$ and even if $\theta$ is irrational, you’ll end up with slices of at most three different sizes! This surprising result is known as the Three Gap Theorem.

If we let our cake have unit circumference and angle $\theta$ has associated arc length $a$, then the above process is like travelling along a 1-periodic sawtooth wave making marks at $a, 2a, \ldots, Na$. In our research, we study generalizations of the Three Gap Theorem to other periodic functions. We investigate the distinct gap sizes created by evaluating periodic functions at uniformly spaced inputs, marking these values, and studying the distinct gap sizes between nearest marks in the image. In particular, we study periodic piecewise linear and trigonometric functions. (Received September 25, 2018)