Periodicity of Certain Generalized Continued Fractions.

We have previously considered continued fractions with “numerator” a positive integer $N$, which we refer to as $\text{cf}_N$ expansions. In particular, let $E$ be a positive integer that is not a perfect square. For $N > 1$, $\sqrt{E}$ has infinitely many $\text{cf}_N$ expansions. There is a natural notion of the “best” $\text{cf}_N$ expansion of $\sqrt{E}$. We have conjectured, based on extensive numerical evidence, that such a best expansion is not always periodic. From this evidence, it is difficult to predict for which $N$ this expansion will be periodic. We show here that for any such $E$, there are infinitely many values of $N$ for which this expansion is indeed periodic, more precisely, periodic of period 1 or 2, and we obtain formulas for a subset of these expansions in terms of solutions to Pell’s equation $x^2 - Ey^2 = 1$. (Received September 03, 2018)