A subset of the integers larger than 1 is \textit{primitive} if no member divides another. Erdős proved in 1935 that the sum of $1/(a \log a)$ for $a$ running over a primitive set $A$ is universally bounded over all choices for $A$. In 1988 he asked if this universal bound is attained for the set of prime numbers. In this paper we make some progress on several fronts, and show a connection to certain prime number “races” such as the race between $\pi(x)$ and $\text{li}(x)$. (Received September 11, 2018)