Call \((p,q)\) a Pythagorean pair if \(p\) and \(q\) are positive integers such that \(p^2 + q^2\) is a perfect square. Draw a line \(\ell\) from the origin into the first quadrant of the \(xy\)-plane. Suppose we want \(\ell\) to avoid all but finitely many Pythagorean pairs with as large a margin as possible. What is the greatest possible margin? What is the second greatest?

In 2008, Romik used a certain ternary tree consisting of Pythagorean triples to define a dynamical system on the unit quarter circle. We will study a Lagrange spectrum arising from Romik’s dynamical system. This provides a natural setting for intrinsic Diophantine approximation on the unit circle. Our result gives a complete answer to the questions posed above. In addition, we obtain an analogue in this context to a classical theorem on Lagrange and Markoff spectra, which was first proved by Markoff in 1879. (Received September 13, 2018)