Let $G$ be a $p$-group and let $\chi$ be an irreducible character of $G$. The codegree of $\chi$ is given by $|G : \ker(\chi)|/\chi(1)$. The set of codegrees of the irreducible characters of $G$ is denoted $\text{cod}(G)$. If $|\text{cod}(G)| = 4$, then $G$ has nilpotence class at most 4 whenever $G$ either has coclass at most 3, largest character degree $p^2$, or $|G : G'| = p^2$. Similar conditions exist which guarantee the existence of $p^2$ as a codegree of $G$. If $|G| = p^{n+1}$ then $\text{cod}(G)$ contains all powers of $p$ up to $p^n$ if and only if $G$ satisfies one of three cases, including the case when $G$ has maximal class and two character degrees. If $G$ has maximal class and $|G|$ is large enough, then $p^3$ and $p^4$ are in $\text{cod}(G)$. The codegrees of maximal class $p$-groups which are also metabelian or normally monomial are always consecutive powers of $p$. The question arises whether all maximal class $p$-groups have consecutive $p$-power codegrees. (Received August 27, 2018)