We set up a framework to study one-dimensional heat equations defined by fractal Laplacians associated with self-similar measures with overlaps. We show that for a class of such self-similar measures, a heat equation can be discretized and the finite element method can be applied to yield a system of linear differential equations. We show that the numerical solutions converge to the actual solution and obtain the rate of convergence. We also study some properties of the solutions of the heat equation. (Received September 24, 2018)