Solving for the fundamental solution to the heat equation on a bounded domain is a classical problem in partial differential equations. When the domain is the circle, for instance, the fundamental solution of the heat equation can be described by a theta function. We talk about using a differential-difference operator $\frac{\delta}{\delta t} - \Delta$ with $\Delta$ the combinatorial Laplacian to model the heat equation on a finite graph analogue of Poincare’s upper half-plane.

Finite analogues of the classical theta functions are shown to be solutions to the heat equation in this setting. The solutions involve zonal spherical functions which come with a natural periodicity. In addition, the related theta functions are automorphic forms. The resultant periodicity interweaves representation theory with the heat equation. (Received September 20, 2018)