Grigoriy Blekherman and Lawrence Fialkow* (fialkowl@newpaltz.edu). The core variety and representing measures in the truncated moment problem.

The classical Truncated Moment Problem asks for necessary and sufficient conditions so that a linear functional $L$ on $P_d$, the vector space of real $n$-variable polynomials of degree at most $d$, can be written as integration with respect to a positive Borel measure $\mu$ on $\mathbb{R}^n$. We work in a more general setting, where $L$ is a linear functional acting on a finite dimensional vector space $V$ of Borel-measurable functions defined on a $T_1$ topological space $S$. Using an iterative geometric construction, we associate to $L$ a subset of $S$ called the core variety, $CV(L)$. The main result is that $L$ has a representing measure $\mu$ if and only if $CV(L)$ is nonempty. In this case, $L$ has a finitely atomic representing measure, and the union of supports of all such measures is precisely $CV(L)$. We also prove a version of the Truncated Riesz-Haviland Theorem in this general setting, and use this to solve the generalized Truncated Moment Problem in terms of positive extensions of $L$. These results are adapted to derive a Riesz-Haviland Theorem for a generalized Full Moment Problem and to obtain a core variety theorem for the latter problem. (Received September 01, 2018)