We consider the bilinear inverse problem of recovering two vectors, $\mathbf{x} \in \mathbb{R}^L$ and $\mathbf{w} \in \mathbb{R}^L$, from their entrywise product. We consider the case where $\mathbf{x}$ and $\mathbf{w}$ have known signs and are sparse with respect to known dictionaries of size $K$ and $N$, respectively. Here, $K$ and $N$ may be larger than, smaller than, or equal to $L$. We introduce $\ell_1$-BranchHull, which is a convex program posed in the natural parameter space and does not require an approximate solution or initialization in order to be stated or solved. We study the case where $\mathbf{x}$ and $\mathbf{w}$ are $S_1$- and $S_2$-sparse with respect to a random dictionary and present a recovery guarantee that only depends on the number of measurements as $L \geq \Omega(S_1 + S_2) \log^2(K + N)$. We also introduce a variant of $\ell_1$-BranchHull for the purpose of tolerating noise and outliers and show it can recover piecewise constant behavior from real images. (Received September 24, 2018)