Yizhen Chen* (johnson.chen@prismus.org). The behavior of iterations of compositions of inversions preserving a circle.

Let $C$ be a circle, and $E$ be a conic. Let $f_E : C \to C$ be a homeomorphism such that line $xf(x)$ is always tangent to $E$ for $x \in C$. Poncelet’s porism states that if $f^n$ has a fixed point, then $f^n$ is the identity. We replace $E$ by a polygon, and study the behavior of a composition of inversions $f_A : C \to C$ where $A$ is a point and line $xf_A(x)$ always passes through $A$ for $x \in C$. We found two different ways to convert a composition $f$ of several inversions into a composition of two inversions. When $f$ has no fixed points, we give a simple condition that $f^n$ has a fixed point ($n > 1$), which is also equivalent to that $f^n$ is the identity. When $f$ has fixed points, one of the fixed points $P$ has the property that $\lim_{n \to \infty} f^n(x) = P$ for every $x \in C$ except the other fixed point. We found a simple criterion to determine which fixed point has this property for a composition $f$ of $m$ inversions $f_{A_k}$. For $f = f_C \circ f_B \circ f_A$ we have another simple criterion when all of $A, B,$ and $C$ are inside $C$. Lastly, there is a conic $E(A, B)$ such that $f_C \circ f_B \circ f_A$ has no fixed points if and only if $C$ is inside it. We found several interesting properties of $E(A, B)$. (Received August 19, 2018)