We propose a general technique related to the polytopal Sperner lemma for proving old and new multilabeled versions of Sperner’s lemma. A notable application of this technique yields a cake-cutting theorem where the number of players and the number of pieces can be independently chosen. We also prove multilabeled versions of Fan’s lemma, a combinatorial analogue of the Borsuk-Ulam theorem, and exhibit applications to fair division and graph coloring. (Received September 24, 2018)