
On a closed Riemannian manifold with nonnegative Ricci curvature, using a gradient estimate of the eigenfunctions of the Laplace-Beltrami operator, Zhong-Yang proved a sharp lower bound for its first non-zero eigenvalue $\lambda_1$. The pointwise lower bound on the Ricci curvature is an assumption that comes from the derivation of the gradient estimate, which makes use of Bochner’s formula.

Recently, similar gradient estimates have been obtained on manifolds where the pointwise lower bound has been replaced by an $L^p$ bound on the negative part of the Ricci curvature. In this talk, we will see how these techniques can be applied to obtain a Zhong-Yang lower bound on manifolds with small negative Ricci curvature in the $L^p$ sense, where the lower bound only depends on geometric parameters and recovers the classical Zhong-Yang lower bound in the limit $\text{Ric} \geq 0$. (Received September 15, 2018)